A Mathematical View of Losing Weight

It will bring us more fun if we use the mathematical method to study the process of losing weight, many overweight people who are losing weight might be vexed by these problems: why my weight regained as soon as I stop dieting and exercising? Why my weight doesn't lose any more even though I'm still dieting and exercising? In this paper, we explained the secret behind these phenomena, and interpreted the essence of wight loss by a mathematical way.

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We will learn some quite profound aspects in losing weight if we use the mathematical method to analyse it. The routine weight loss strategy for obese people is to diet and exercise in a certain period, they may struggle with the hunger and their free will in that period, normally they will get some accomplishment after going through this hard time, but a lot of people might have some problems such as, why my weight regained as soon as I stop dieting and exercising? And, why my weight doesn't lose any more even though I'm still dieting and exercising? Don't worry, you will find the answer after reading this article, and we firmly believe that you will gain a completely new perspective in the weight loss.

1. How did you gain fat?

"It really annoys me when I'm getting on a scale and find that I am getting fat again!" So why would you get fat?

The fat is made of carbon, oxygen and hydrogen, but it is not able to synthesize it only with these materials, the process needs the energies supplied. And where will these energies come from? They come from the food you eat.

When you purchase the food from a supermarket, there will be a nutrition information format on the packing bag, for example, the following format is the nutrition information of a box of milk:

FACTORS	CONTENT/100ML	VALUE
ENERGY	139 kj	2%
PROTEIN	3.2 g	5%
FAT	0 g	0%
CARBOHYDRATE	5.0 g	2%
SODIUM	60 mg	3%
CALCIUM	105 mg	13%

The "Energy" item in the format above is the energy the food can provide, the unity of the energy is "kilojoule", but what we usually heard is another familiar item, "calories", which is just another name of the energy unit, the connection between them is simple, 1000 calories = 4.128 kj, we shall use 1 k.cal to represent 1000 calories. For example, a box of milk is about 250 ml, thus the total calories it can provide is 82 k.cal.

Normally, eating is the only way we can gain the energies, meanwhile, a lot of our daily activities will consume the energies, such as walking, jogging, playing tennis and some other physical exercises, also our brain will consume a lot of energies when we are thinking some hard problems, maths for instance, even when you are watching a horror movie, it consumes some calories, however, the most important consumption is the basal metabolism.

Basal metabolism (BM) is described as the total amount of heat in kilocalories that the organism produces during a certain time (a day, for instance). Roughly speaking, the BM is the calories you will consume when you sleep on an extremely soft bed whole day. This part of consumption will account for 60%~80% of person's daily consumption! And the fatter the person, the bigger the proportion. There is a simple formula in nutriology to compute the BM, for female, the BM is

$$\begin{aligned}\mathrm{BM} = {} & 9.6 \times \text{weight} + 1.72 \times \text{height} \\ & - 4.7 \times \text{age} + 661\end{aligned}$$

And for male is

$$\begin{aligned}\mathrm{BM} = {} & 13.73 \times \text{weight} + 5 \times \text{height} \\ & - 6.9 \times \text{age} + 67\end{aligned}$$

For example, a 20-year-old boy Tom, with 180 cm tall and 70 kg weight, his daily BM is 1790.1 k.cal.

In fact, the BM is also related to the proportion of muscles, normally, people with more muscles will have lager BM amounts, but in this paper, we will neglect this meticulous factor.

Let the calories you gained minus the calories you burned, the rest, if any, will be used to synthesize the fat, from the experiments of biology, we can know that it needs 9000 k.cal to produce 1 kg fat. For example, the boy Tom above, who usually eats 2200 k.cal food, but he rarely exercises, the whole calories he will consume per day is 2000 k.cal, then he will gain $200/9 \approx 22.2$ g fat every day, and day after day, he will become fatter and fatter.

2. Will your weight grow endlessly?

Do you have such a worry? I eat so much food every day, my weight will eventually break through 500 kg! Will this worry come true?

Well, before we answer this question, we need to know a fact that everyone will have an eating habit and an exercising habit, that is to say, one must eat so much food every day, if less, one may feel hungry, if more, one will feel full and uncomfortable, the same explanation for exercising habit.

Now, let's restate the problem at the beginning of this section, if you eating too much every day but regularly, your weight will grow, but meanwhile, your BM is also growing, so, will your weight grow endlessly?

The answer is surprisingly NO! We shall use the mathematical method to study this phenomenon rationally, so let's do the math right now.

We shall only study the case for male, since for female the calculation is the same. If we denoted by x k.cal , the calories of food you eat every day, and y k.cal the calories

you burned in physical exercises every day, and $\Delta = x - y$ the difference of those calories, and we assume the weight function about the days t is $m(t)$, to be convenient, we denoted by $Q(t)$ the BM function, and for male the function is

$$Q(t) = 14m(t) + C$$

Where C is a constant which equals to $67 + 5 \times \text{height} - 6.9 \times \text{ages}$. Since the increasement of the fat is converted by the extra calories, so we can have a mathematical description of this process, that is an integral equation:

$$\int_0^t (\Delta - Q(\tau))\,\mathrm{d}\tau = 9000\,(m(t) - m(0))$$

If in addition, we suppose the weight changes smoothly, i.e., $m(t)$ is a smooth function, we will have a differential equation:

$$\Delta - C - 14m = 9000m'$$

We can solve this equation immediately:

$$m(t) = \left(m(0) - \frac{\Delta - C}{14}\right)e^{-\frac{7t}{4500}} + \frac{\Delta - C}{14}$$

It is clear from this solution that your weight will not grow endlessly! It will at the level of $(\Delta - C)/14$ kg eventually!

For example, the boy Tom we mentioned before, whose eating habit is 2200 k.cal per day and exercising habit is about 210 k.cal per day, he will become fat, but at $(2090 - 829)/14 \approx 82.9$ kg tops.

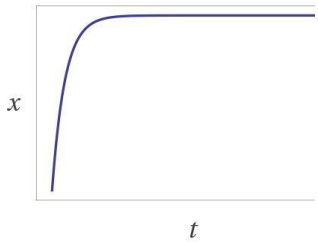

Figure 1: The weight changing function

So, don't worry! As long as you eat regularly, your weight will eventually be stable even if you eat a lot.

3. What is the essence of weight loss?

As it has been shown above, the weight is dominated by the eating habit and exercising habit, we will finally get a stable weight that compatible with our habits. Now, it is not hard to answer the questions at the beginning.

Why my weight always regained as soon as I stop dieting and exercising? Because you recover your original living habits, hence the weight will be naturally regained back.

Why my weight doesn't lose any more even if I'm still dieting and exercising? Because you probably have already attained your limit weight which compatible with your current eating habit and exercising habit.

Thus, the root cause of the obesity is our eating habit and exercising habit changed to be unhealthy, and what is the essence of weight loss?

The essence is to **develop a healthy eating habit and exercising habit!** If you have owned a healthy habit successfully, then the weight decreasing to a desired amount is just a problem of the time, and who tells us this philosophy? Mathematics!

4. Further Reading

D.M. Thomas; M. Scioletti and S.B. Heymsfiel, "Predictive Mathematical Models of Weight

Loss". *Curr Diab Rep*.**19**, 93(2019).