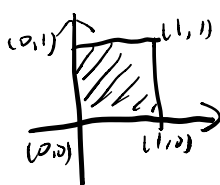
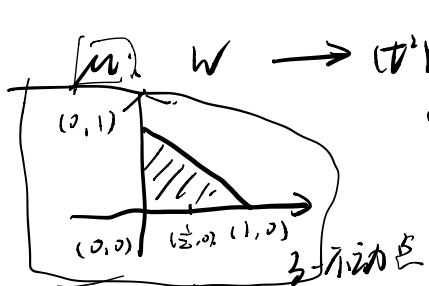


Exercise 13. 连通.

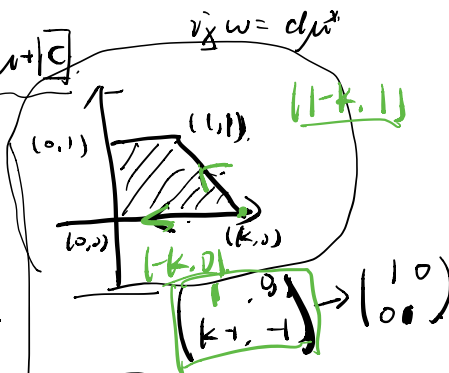
分类 $\left\{ \begin{array}{l} W-4 \text{ 维辛流形 (有效 Hamilton } T^2\text{-作用)} \\ \text{不超过 4 个不动点} \end{array} \right.$

symplectic toric Defant Thm. (高平移)

By Moment map



$$W' = W + [C]$$



① $\mathbb{CP}^2$

② $\mathbb{CP}^1 \times \mathbb{CP}^1$

③ Hirzebruch surface. (Ex IV. 4)

$$\mathbb{T}^2 \times \mathbb{CP}^2 \rightarrow \mathbb{CP}^2$$

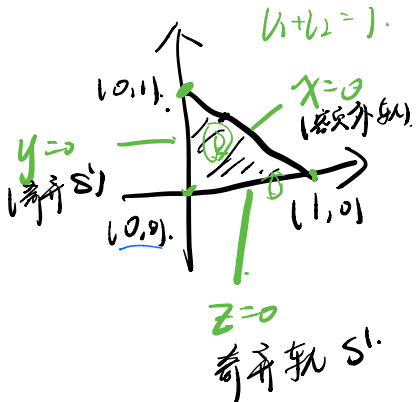
$$((u, v), ([x, y, z])) \rightarrow (x, u^2 y, v^2 z)$$

$$\mu = \frac{1}{2} \left(\frac{2|y|^2}{|x|^2 + |y|^2 + |z|^2}, \frac{2|z|^2}{|x|^2 + |y|^2 + |z|^2} \right)$$

$$= \left(\frac{|y|^2}{|x|^2 + |y|^2 + |z|^2}, \frac{|z|^2}{|x|^2 + |y|^2 + |z|^2} \right)$$

$$X = \left(\frac{\partial}{\partial t_1}, \frac{\partial}{\partial t_2} \right) \quad \Delta = \left(2 \frac{\partial}{\partial z}, 2 \frac{\partial}{\partial \bar{z}} \right)$$

$$\begin{array}{ccc} \mathbb{T}^2 \times \mathbb{CP}^1 & \rightarrow & \mathbb{CP}^1 \\ \downarrow \pi & & \downarrow \pi \\ \mathbb{T}^2 \times \mathbb{CP}^1 & \rightarrow & \mathbb{CP}^1 \end{array}$$



$$\mathbb{CP}^2$$

$$\begin{Bmatrix} [1, 0, 0] \\ [0, 1, 0] \\ [0, 0, 1] \end{Bmatrix} \text{ 不动点 }$$

polytope.

$$\begin{Bmatrix} (0, 0) \\ (1, 0) \\ (0, 1) \end{Bmatrix}$$

$$|y|^2 = |x|^2 + |y|^2 + |z|^2$$

$$[1, y, 0]$$

$$z=0$$

$$k|y|^2 = |x|^2 + |y|^2 + \frac{a}{k}$$

$$(k-1)|y|^2 = |x|^2$$

$$|y|^2 = \frac{|x|^2}{k-1} = \frac{1}{k-1}$$

$$\mu^{-1}(k, 0) = S^1$$

$$T^2 \times \mathbb{CP}^2 \rightarrow \mathbb{CP}^2$$

$$(u, v), [1, y, 0] \rightarrow [1, u^2 y, 0]$$

稳定化子是 $u = \{e^{2\pi i}\}$ v 任意 $\cong \{e^{2\pi i}\} \times S^1$

轨道 $T^2 / \text{稳定化子} \cong S^1 \times S^1 / \{e^{2\pi i}\} \times S^1 \cong S^1$

奇异轨道 $(\dim S^1 = 1)$

$$x=0. [0, y, z] \rightarrow [0, u^2 y, v^2 z]$$

稳定化子 $u = e^{2\pi i} \quad v = e^{2\pi i} \quad (e^{2\pi i}, e^{2\pi i})$

轨道 $\cong S^1 \times S^1 / (e^{2\pi i}, e^{2\pi i}) \cong S^1 / e^{2\pi i} \times S^1 / e^{2\pi i}$
 $\cong S^1 \times S^1 \cong T^2$

$$\dim T^2 = 2$$

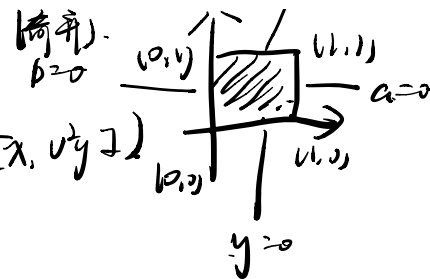
Ex 3. $(u, v) [x, y, z] = [x, uy, v^2 z]$

计算: $\mu^{-1}(x, y, z) = \frac{1}{2} \left(\frac{|y|^2}{|x|^2 + |y|^2 + |z|^2}, \frac{2|z|^2}{|x|^2 + |y|^2 + |z|^2} \right)$

x=0

$$\textcircled{2} \quad T^2 \times (\mathbb{C}P^1 \times \mathbb{C}P^1) \rightarrow \mathbb{C}P^1 \times \mathbb{C}P^1$$

$$((u, v), ([a, b], [\lambda, \eta])) \rightarrow ([a, u^2 b], [\lambda, v^2 \eta])$$



$$\mu = \frac{1}{2} \left(\frac{2|b|^4}{|a|^2 + |b|^2}, \frac{|b|^2}{|a|^2 + |b|^2} \right)$$

$$\mathbb{C}P^1 \times \mathbb{C}P^1$$

polytope

$$([1, 0], [1, 0])$$

$$[0, 0]$$

$$([0, 1], [1, 0])$$

$$[1, 0]$$

$$([1, 0], [0, 1])$$

$$[0, 1]$$

$$([0, 1], [0, 1])$$

$$[1, 1]$$

$$([a, b], [\lambda, 0])$$

$$y=0$$

↓

$$[a, u^2 b], [\lambda, 0]$$

$$\text{稳定化子 } u = \{e^{2\pi i}\}$$

$$v \text{ 任意 } \simeq S^1$$

$$\text{轨道 } S^1 \times S^1 / \{e^{2\pi i}\} \times S^1 \simeq S^1$$

奇奇